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Calculating the Self-Aligning Torque of Agricultural Tyres on Firm Surfaces

Tyres play a very important role in the driving dynamics of agricultural vehicles, since they are the direct link between the vehicle and the ground. Thus, an exact tyre model is essential for driving dynamics simulation. For this purpose, a transient tyre model was developed at the University of Hohenheim during recent years. It can be used for calculating the forces and torques acting on the tyre. Especially for the steering design, the self-aligning torque of the tyre is of special interest. In this article the calculation methods and some validating results are presented.

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Keywords

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Literature

Literature references can be called up under LT 07314 via internet http://www.landwirtschaftsverlag.com/landtech/local/literatur.htm.

In the design process of passenger cars or trucks different simulation tools are widely used for driving dynamics analysis. Thus, various tyre models optimized for these applications are available on the market. However, very few models exist for soft agricultural tyres. Therefore at the University of Hohenheim a transient tyre model for agricultural tyres was developed. It calculates the transient forces and moments acting on the wheel. While the forces play an important role in the vehicle's handling, the self-aligning torque has a strong influence on the steering.

Modelling of the self-aligning torque

While the Hohenheim Tyre Model calculates the forces as a point contact [2], for the calculation of the self-aligning torque the force distribution in the contact patch is needed. A simple model for small slip angles is used by Strackerjan [7], while Holtschultze [4] and Gim [3] used an improved approach. According to Kabe [5], the self-aligning torque has three causes: shear deformation of the rubber, lateral deflection and torsion of the tyre around its vertical axis. The Hohenheim Tyre Model uses the shear deformation in the contact patch and the torsion of the tyre to calculate the self-aligning torque. The contact patch at combined slip is depicted in *Figure 1*. The resulting force *F* acts in centre *s* of the grey area, which represents the force distribution. The velocity of the wheel is here divided into a longitudinal and a lateral component, v_{tx} and v_y.

For the calculation of the shear stress, the assumed movement of a tread particle along the path *p* is considered. The particle enters the contact patch in its plane of symmetry and moves along *p* according to the slip angle of the contact patch α_{st}. In the area A_1, it is assumed that the rubber adheres to the ground, while in the area A_2 a sliding of the tread occurs. The length l_{t1} of the adhesion area depends on the longitudinal and lateral sliding velocity of the tread and is calculated as follows:

$$l_{t1} = l_t - (|\tan\alpha_{st}| + |\sigma_{st}|) \cdot l_t \qquad (1)$$

It is assumed that the length of the sliding area can be described by the ratio between the lateral sliding speed and the longitudinal speed - $\tan\alpha_{st}$. The second part of the sliding area depends on the longitudinal slip α_{st}. The entire tread length l_t is calculated using Pythagorean Theorem:

$$l_t = 2 \cdot \sqrt{r_{constr}^2 + r_l^2} \qquad (2)$$

Where: r_{constr} – tyre design radius and r_l – distance between the wheel hub and the ground.

The lever arm of the lateral force is represented by the distance s_x between center of gravity *s* and the axis of symmetry of the tyre tread and is calculated in the equation (3).

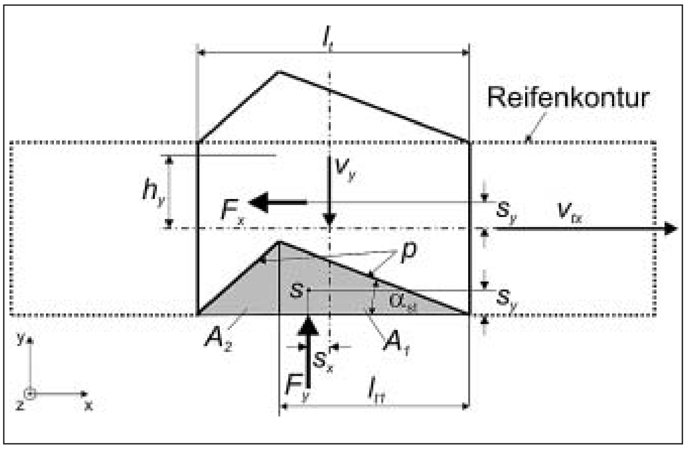

Fig. 1: Strongly simplified top view of a tyre tread with force distribution

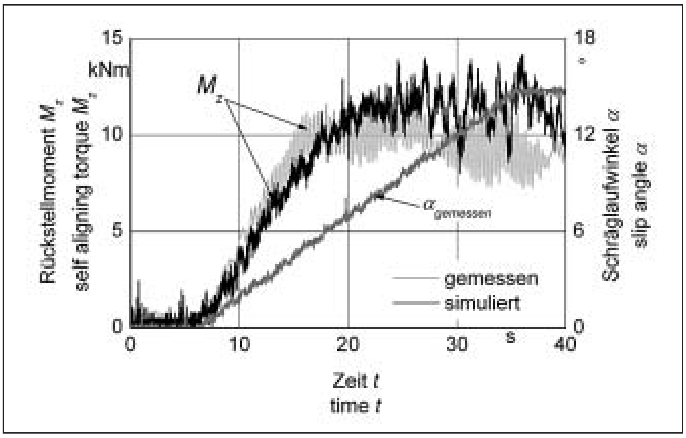

Fig. 2: Self-aligning torque of a pulled wheel at a slip rate angle of α′ = 0.5°/s, a static wheel load F_z = 20 kN and v_{tx} = 2 km/h

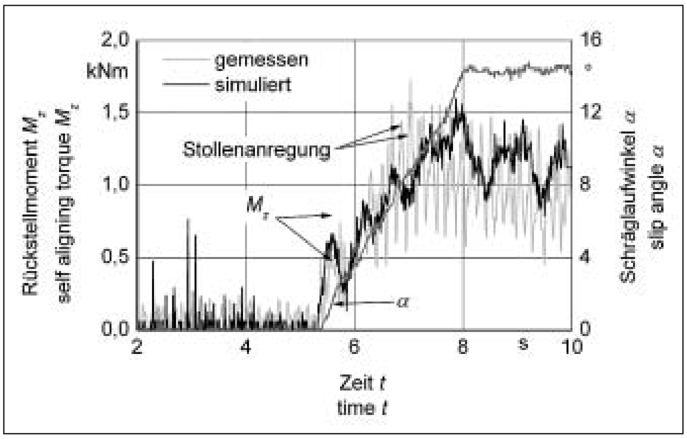

Fig. 3: Self-aligning torque of a pulled wheel at a slip rate angle of α′ = 5°/s, a static wheel load F_z = 20 kN and v_{tx} = 5 km/h

((Gleichung einsetzen)) (3)

The lever arm of the longitudinal force is given as:

((Gleichung einsetzen)) (4)

Subsequently, the component of self-aligning torque due to the shear force in the tyre tread can be calculated as follows:

$$M_{scher} = F_y \bullet s_y - F_x \bullet s_x \quad (5)$$

The second component of the self-aligning torque is caused by the torsion of the tyre around its vertical axis. The slip angle α of the wheel hub differs from the slip angle of the tread α_{st} [2]. This difference is the angle of vertical tyre torsion $\Delta\alpha = \alpha - \alpha_{st}$, which can be used as an input for a torsional Voigt-Kelvin-Element. According to this approach, torsion occurs only at existing slip angles. Thus, a calculation of the self-aligning torque for a standing wheel is not possible yet. However, an extension concerning this matter is imaginable.

To define the Voigt-Kelvin-Element, a torsional stiffness c_{tors} and damping d_{tors} are needed.

The corresponding spring-damper-equation leads to the torsion-caused component of the self-aligning torque:

$$M_{tors} = c_{tors} \bullet \Delta\alpha + d_{tors} \bullet \Delta\dot{\alpha} \quad (6)$$

Finally, the complete self-aligning torque is composed of both components as follows:

$$M_z = M_{scher} + M_{tors} \quad (7)$$

Simulation results

The calculation of the self-aligning torque was verified on the single-wheel tester. This test rig allows measurements of the forces and torques acting on a pulled or driven wheel, which at the same time can be steered [1; 6]. The following results show a 520/70 R 38 tyre at an inflation pressure of 1.2 bar. Since the front wheels are usually not driven at higher speeds, the focus lies here on a pulled wheel. *Figure 2* shows the self-aligning torque during a quasi-stationary change of the slip angle. Obviously, the Hohenheim Tyre Model achieves quite high accuracy at slip angles up to 6°. Slip angles higher than 6° are already relatively high values, seldom reached in normal driving situations.

The transient behaviour is depicted in *Figure 3*. Also the transient behaviour at a slip angle rate of 5 °/s can be simulated using the Hohenheim Tyre Model. Deviations occur at larger slip angles, which is analogue to the steady state case. Furthermore, the influence of the tyre lugs can be observed. However, these lug-induced vibrations get too high-frequencies at higher driving speeds, so they do not have a significant influence on the driving behaviour. Thus, they are neglected in the Hohenheim Tyre Model.

Another important case is the self-aligning torque at changing adjustment direction of the slip angle (*Fig. 4*). The slip angle is increased up to 14° and finally decreased to zero. It is adjusted at a rate of 10 °/s. The calculated torque matches the measured value quite well, although the simulation result is slightly too dynamic around the maximum slip angle.

Conclusion

The hybrid concept of the Hohenheim Tyre Model allows a fast calculation of the relevant driving dynamics values on rigid surfaces. Like for the forces, one of the aims for the torque calculation was a low number of parameters. Beside that, a physical approach was considered important for the calculation of the transient behaviour. The principles of the force calculation of the Hohenheim Tyre Model were used for the calculation of the self-aligning torque as well. Subsequently, only two additional parameters were needed for the calculation of the self-aligning torque. Although some slight deviations occur at slip angles above 6°, it can be concluded that the calculation of the transient self-aligning torque using the Hohenheim Tyre Model leads to quite good accordance with measurement results.

Fig. 4: Self-aligning torque of a pulled wheel during a sign change of the slip rate angle at a wheel load of F_z = 20 kN and v_{tx} = 2 km/h